On the Golden Rule of Trike Design

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Synopsis—The Golden Rule of Trike Design

To ensure stability of a trike with two wheels on the front axle, the centre of gravity (centre of mass) must be a third of the wheelbase back from the front axle. This gives a static vertical wheel load of a third of the weight on each wheel; equal to 66% on the front axle and 33% on the rear.

Abstract

Huston, Graves and Johnson's SAE paper, 'Three Wheeled Vehicle Dynamics'[1] appears to be the initial source for the *golden rule of trike design* (or *66/33 rule*), which prescribes a very limited area for the location of the centre of gravity for a tadpole trike.

In this article, I suggest that this rule is overly limiting as a design criteria, and that the "golden rule" might be better replaced by three design guidelines.

1 Introduction

Please note that this is article is a review of, and discussion about, published findings. Investigation, debate, disagreement and reasoning are all normal parts of the scientific method, and should not *be construed to be a personal attack on anyone. In a similar vein, please contact the author (above) with any questions, clarifications, improvements or counter-arguments.*

The Huston, Graves and Johnson's (HGJ's) paper[1] on three-wheeled vehicle dynamics looked at two components of vehicle stability, namely *lateral stability* and *rollover stability.* It includes analysis of four-wheeled vehicles and three-wheeled vehicles with two wheels on the front axle (tadpole configuration) and two wheels on the rear axle (delta configuration).

1.1 Definitions

Lateral Stability Depending on the value of the understeer coefficient a vehicle may be either directionally stable at all speeds, or become unstable above a threshold speed (known as the *critical speed*).

Rollover Stability A measure of a vehicle's tendency to tip over sideways when a lateral body force is applied (e.g. during cornering).

2 Review of Rollover Stability

The rollover stability calculations differ slightly in method for, but come to to the same conclusion as, an article I wrote recently on tricycle stability (`http://www.deferredprocrastination.co.uk/blog/2010/tricycle-stability`). In the paper though, the authors went further then I, and calculated the maximum cornering velocity of a standardised model for each wheel arrangement with a selection of centre of gravity positions and for cornering with no change of speed, cornering with 0.25g braking and cornering with 0.25g acceleration.

The four-wheeled model was shown to have a consistent rollover speed for all centre of gravity and acceleration conditions; indicating that centre of gravity position along the centreline does not affect rollover stability in this case. The paper also concludes that if the vehicle's track is greater than twice the height of the centre of gravity, the vehicle will "always slide laterally before overturning."[1]

The rollover speed for the three-wheeled vehicle is shown to vary with longitudinal acceleration, depending on both configuration and centre of gravity position. Rollover stability is increased when the centre of gravity is closer to the two-wheel axle, and when braking though a corner for the tadpole, and accelerating with cornering for the delta. In the conclusion, the authors state that though the stability of the three-wheeled vehicle is lower than the four-wheeled vehicle, "...the closer the mass center is to the axle with two wheels, the more stable each vehicle becomes with regard to rollover."[1]

3 Review of Lateral Stability

The golden rule of trike design appears to stem from HGJ's calculations on lateral stability. The authors conclude that, "To ensure lateral stability for the situation of constant speed straight line motion, it is recommended that the three wheeled vehicle with two wheels on the front axle be designed so that its mass center is located in the front third of the vehicle..."[1]

Just to clarify, this statement relates to lateral stability only; this is *completely separate* from their calculations of rollover stability.

The understeer coefficient (a measure of lateral stability) describes the relationship between the front and rear tyre *slip angles* and is a characteristic of the vehicle.

The slip angle of a tyre is the difference between it's direction of rotation and its direction of travel, due to the deformation of the tyre as it moves through the contact patch. It is this deformation that gives rise to the lateral tyre forces that allow turning.

For more information on tyre behaviour, take a look at the wikipedia page about slip angles or, try a book such as Race Car Vehicle Dynamics*, Milliken Milliken).*

If understeer coefficient is,

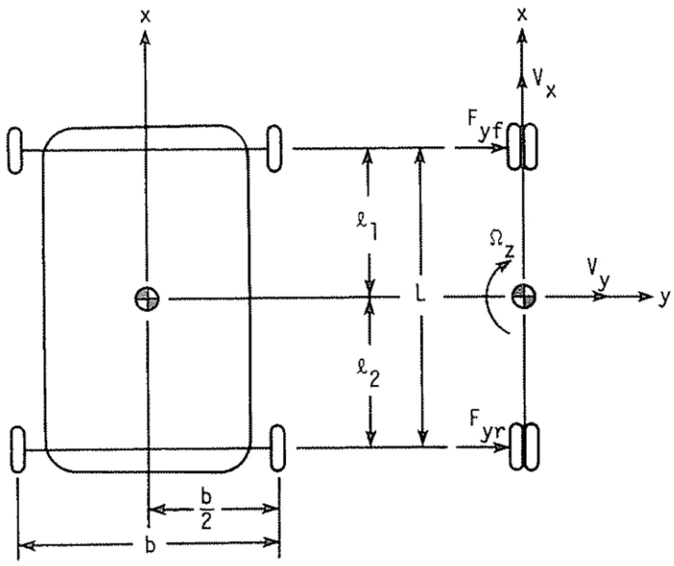

Figure 1: "Four wheeled vehicle model"[1]

- greater than zero: the tyre slip angles are greater at the front than at the rear. This is a dynamically stable situation at all speeds, because the the yaw response of the vehicle to steering reduces, as speed increases.
- equal to zero: the slip angles will be identical at the front and rear axles. Yaw response to constant steering will be unaffected by vehicle speed.
- less than zero: slip angles at the rear will be larger than the front. For any steering angle, the yaw (turning) response will be greater as speed increases; this situation becomes unstable once the vehicle reaches it's *critical speed.*

Understeer in this sense does not relate to vehicle handling at the limit of tyre performance, it refers to the tyre's performance in every turning condition. It might be helpful to think of this effect as over- or under- *cornering* when compared to a theoretically neutral cornering response.

To determine the effect of weight distribution on the understeer coefficient, and therefore lateral stability, HGJ's paper uses a *single track model.* This will give a first approximation of understeer gradient, though slip angles will also be affected by tyre and suspension characteristics. I've reproduced the diagrams for the four-wheeled (figure 1) and three-wheeled tadpole (figure 2) vehicles here; figure 1 and 3 from the paper respectively.

The single track model is a mathematical simplification of a three dimensional vehicle into a one dimensional model. The width and height of the vehicle to reduced to zero, and all tyres forces are assumed to act together at a single point that is the total for the whole axle. From The Bosch Automotive Handbook[2] (chapter: Influences in Motor Vehicles, section: Dynamics of Lateral Motion) a single track model assumes:

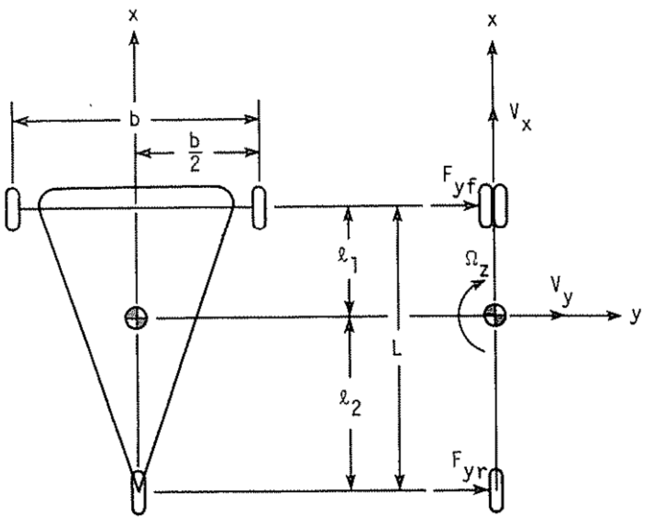

Figure 2: "Three wheeled vehicle model with two wheels on the front axle"[1]

- Kinematics and elastokinematics of the axles are considered only in the linear form.
- The lateral force structure of the tyre is linear and the aligning force structure of the tyre is linear and the aligning of return torque of the tyre is ignored.
- The centre of gravity is at the level of the road surface. This means the vehicle executes only the yaw motion as a rotational degree of freedom. *Roll, pitch and lift are not taken into consideration.* [emphasis added]

Using the nomenclature from the paper:

$$
\begin{aligned}
m &= \text{total mass of the vehicle} \\
V_x &= \text{speed in the } x \text{ direction} \\
V_y &= \text{speed in the } y \text{ direction} \\
\Omega_z &= \text{rotational (yaw) speed about the vertical axis} \\
C_{\alpha f}, C_{\alpha r} &= \text{cornering stiffness per tyre on the front and rear axle respectively} \\
L &= \text{wheelbase (equal to } l_1 + l_2\text{)} \\
g &= \text{gravitational constant} \\
W &= \text{weight of the vehicle} \\
W_f, W_r &= \text{weight (vertical load) on each tyre, for the front and rear axle respectively} \\
F_{yf}, F_{yr} &= \text{Lateral forces on the front and rear tyres (respectively)} \\
K_{us} &= \text{understeer coefficient} \\
A, B &= \text{positive constants dependent upon the tyre properties}
\end{aligned}
$$

HGJ demonstrate resolving the equations of motion for the four-wheeled vehicle so they can determine the equation for the critical speed (V_{crit})and understeer coefficient (K_{us}).

$$V_{crit} = \sqrt{\frac{-gL}{K_{us}}} \tag{1}$$

and,

$$K_{us} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \tag{2}$$

Cornering stiffness is a combination of tyre properties and vertical load:

$$C_\alpha = (A - BW_t)W_t \tag{3}$$

By calculating the static tyre loadings, they then solve the equation for understeer coefficient,

$$W_f = \frac{Wl_2}{2L} \qquad \text{and} \qquad W_r = \frac{Wl_1}{2L} \tag{4}$$

$$
\begin{aligned}
K_{us} &= \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} = \frac{BW(l_2 - l_1)}{2L(A - BW_f)(A - BW_r)} \\
&\geq 0
\end{aligned} \tag{5}
$$

So for K_{us} to always be greater than zero,

$$l_2 \geq l_1 \qquad \text{or} \qquad l_2 \geq \frac{L}{2} \tag{6}$$

Thus, they conclude, if the centre of gravity is in the front half of the wheelbase, the vehicle will have lateral stability at all speeds. That is, the critical speed will be equal to the square root of a negative number, which is a non-real (imaginary) number.

In a similar manner, for the tadpole trike layout, HGJ give the vertical wheel loadings to be:

$$W_f = \frac{Wl_2}{2L} \qquad \text{and} \qquad W_r = \frac{Wl_1}{L} \tag{7}$$

$$\begin{aligned} K_{us} &= \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} = \frac{BW(l_2 - 2l_1)}{2L(A - BW_f)(A - BW_r)} \\ &\geq 0 \end{aligned} \tag{8}$$

So, to ensure stability at all speeds,

$$l_2 \geq 2l_1 \qquad \text{or} \qquad l_2 \geq \frac{2}{3}L \tag{9}$$

Showing that the centre of gravity must be within the front third of the wheelbase.

4 Comment

Having reviewed HGJ's work on lateral stability, there is a query as to the difference in lateral stability between the four-wheel and three-wheeled understeer gradient calculations.

Because the whole point of the single track model is to simplify the kinematics of a vehicle system, it excludes—as HGJ write, "...roll freedom, suspension effects, driver control, time delays, *lateral normal load transfers* or nonlinearities...". [emphasis added]

Because a single track model assumes all tyre forces act at a single point at at the centre of each axle, there is no apparent reason why the formula to calculate K_{us} should be any different for a three or four wheeled vehicle. One, two, three, six or ten wheels per axle should have no effect in equation 2 because the single track model only ever has two "tyres", one representing the cumulative effects at the front axle and the other representing the cumulative effect at the rear axle.

I believe that it is possible there is a confusion in the paper between the use of of *tyre load* and *axle load*. To clarify this, the nomenclature needs expanding slightly:

$$\begin{aligned} W_x &= \text{vertical load} \\ C_{\alpha x} &= \text{cornering stiffness} \end{aligned}$$

Where x may be one of the following suffixes:

$$\begin{aligned}
{}_f &= \text{front} \\
{}_r &= \text{rear} \\
{}_{fl} &= \text{front left} \\
{}_{fr} &= \text{front right} \\
{}_{rl} &= \text{rear left} \\
{}_{rr} &= \text{rear right}
\end{aligned}$$

For a four-wheeled vehicle then,

$$W_{fl} = \frac{Wl_2}{2L} \qquad \text{and} \qquad W_{fr} = \frac{Wl_2}{2L} \tag{10}$$

$$W_f = W_{fl} + W_{fr} = \frac{Wl_2}{L} \tag{11}$$

and,

$$W_{rl} = \frac{Wl_1}{2L} \qquad \text{and} \qquad W_{rr} = \frac{Wl_1}{2L} \tag{12}$$

$$W_r = W_{rl} + W_{rr} = \frac{Wl_1}{L} \tag{13}$$

Note the differences between vertical loads from HGJ (equation 4) and equations 10 thru 13.

Because in the single track model, a single “tyre”represents the properties of all tyres on that axle,

$$C_{\alpha f} = (A - BW_f)W_f \tag{14}$$

$$C_{\alpha r} = (A - BW_r)W_r \tag{15}$$

so K_{us} can be calculated as:

$$\begin{aligned} K_{us} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} &= \frac{BW(l_2 - l_1)}{L(A - BW_f)(A - BW_r)} \\ \geq 0 \end{aligned} \tag{16}$$

Though this shows K_{us} to be twice that from equation 5, there is no difference in the ratio of $l_1 \geq l_2$. Therefore, for a four-wheeled vehicle to be laterally stable at all speeds, the centre of gravity should be in the front half of the wheelbase.

Analysing a tadpole trike layout, using the same, single track model; the front axle load would be,

$$W_{fl} = \frac{Wl_2}{2L} \qquad \text{and} \qquad W_{fr} = \frac{Wl_2}{2L} \tag{17}$$

$$W_f = W_{fl} + W_{fr} = \frac{Wl_2}{L} \tag{18}$$

while on the rear there is only one wheel, meaning,

$$W_r = \frac{Wl_1}{L} \tag{19}$$

and K_{us} can be calculated now as:

$$\begin{aligned} K_{us} &= \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} = \frac{BW(l_2 - l_1)}{L(A - BW_f)(A - BW_r)} \\ &\geq 0 \end{aligned} \tag{20}$$

Showing that a tadpole trike layout will have lateral stability at all speeds if $l_2 \geq l_1$, that is if the center of gravity is in the front half of the wheelbase; not just the front third as Huston, Graves and Johnson suggest. This does assume the same tyre properties front and rear ($A_f = A_r$ and $B_f = B_r$) and it could be shown to be valid for a delta layout too.

And this result is still valid if the single track model is expanded to the include a first approximation to weight transfer during cornering. By using equation 18 to expand equation 14:

$$C_{\alpha f} = (A - B(W_{fl} + W_{fr})).(W_{fl} + W_{fr}) \tag{21}$$

And during any steady state cornering, any vertical load not on the left wheel will be on the right because, as equation 18 shows: the total load on the axle will remain the same.

5 Summary

As a result of the above, I would propose that the "golden rule" might be better replaced with the following three "rules of thumb" as guidance for tricycle design:

1. The centre of gravity should be mounted as close to the two-wheel axle as possible to maximise *rollover stability*.
2. The height of the centre of gravity should be less than half the track measurement (and less than the distance to the front axle).
3. If the centre of gravity is in the front half of the vehicle, the vehicle will be stable at all speeds, otherwise further calculation is necessary to determine the speed limit of *lateral stability* (equation 1).

References

[1] Huston JC, Graves BJ, Johnson DB, *Three Wheeled Vehicle Dynamics*, Society of Automotive Engineers, 820139, 1982

[2] Robert Bosch GmbH, *Bosch Automotive Handbook, 6th Edition*, Professional Engineering Publishing, October 2004

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